MaR-T: Designing a Projection-Based Mixed Reality System for Nonsymbolic Math Development of Preschoolers

Guided by Theories of Cognition and Learning

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ABSTRACT

Recent developmental studies state that nonsymbolic number representation (i.e., more-less comparisons) is important for math development, and children's judgment about such non-numerical magnitudes can be affected by sensory properties (i.e., volume, space). Yet, to our knowledge, there are no tangible based systems for training this math concept. Building on theories of cognition and learning, we developed MaR-T, a projector-camera setup. This paper is a step towards investigating the effects of projection-based mixed-reality (MR) system with tangibles on nonsymbolic number representation of 3- to 5-year-old children. We present our user studies with a total of 14 participants, conducted to observe their interaction with the setup and the possible effects of our design on learning. The results indicate that MaR-T can provide active, engaging, and social learning, and our insights can inspire other interaction design and education studies.

CCS CONCEPTS

• Human-centered computing~ **Mixed/augmented reality** • Human-centered computing~ **User interface design** • Social and professional topics~ **Children**

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Figure 1. MaR-T, interaction with the setup.

KEYWORDS

Tangible interaction; Embodied learning; Nonsymbolic math; Mixed reality; Interaction design for children; Preschool

INTRODUCTION

Early math competency drives a child's later academic achievements and influences employment in STEAM (science, technology, engineering, arts, and math) fields [20,32]. For its development, nonsymbolic number representation (i.e., comparing quantities as more, less or equal) is an important base. In order to formulate an understanding of such concepts, a dialog and reflection on interactions with the physical world are important [2,19,30]. Yet, there are only tablet interfaces to train nonsymbolic math. These applications fall short to convey some of the critical facets of this concept that is affected by perceptual cues (i.e. space, volume) [18,21]. Tangibles, naturally having perceptual cues and allowing spatial reconfigurations, can help children to comprehend this math phenomenon. To our knowledge, there are no tangible based systems that target nonsymbolic math.

On the other hand, developing tangible based training for non-symbolic education is a multifaceted activity. First, a pitfall of physical objects is that resembling everyday items (i.e., cars) help link abstract concepts with the real world, but might prime children to a certain representation that limits generalizability[10,33,40]. To that end, Mixed Reality (MR) setups with projection offer the flexibility to change the perceived properties of the same artifact. Second, manipulating objects without guidance has limited contribution to a taught concept [1].

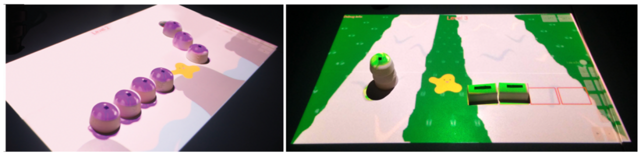
Figure 2. MaR-T: Snapshots from different levels.

So, tangible user interfaces (TUI) can provide audio, visuals and congruent feedback to guide the children. On top of these aspects, to design for effective learning, the pillars of learning (meaningful, active learning, engagement, social interaction) have to be addressed [28].

In this regard, we designed MaR-T, a projection-based MR system with tangibles that aims to train 3- to 5-year-old children on nonsymbolic number representation (Figures 1& 2). Our setup consists of a small projector and a depth camera, which tracks objects and gestures.

The design of MaR-T follows research through design process [65]. The preliminary design of MaR-T was tested with four participants (Mage= 3.8 years) to observe the usability of the system and children' interactions. Our first findings revealed that our setup was usable and could be adapted to support effective learning. In that respect, we developed the second version of the MaR-T. In this version, 1) we aimed to promote *active learning* by asking thought-provoking questions, 2) added new physical interactions to maintain *engagement*, 3) used different spatial reconfigurations in each module towards *meaningful learning*, 4) added parasocial interaction with our narrative character to build *social interaction.*

We conducted our second study with ten children (*Mage*= 4.7 years) to see the effect of our design decisions. Findings reveal that MaR-T has the potential to support the pillars of learning. The results also show that TUI's physical affordances might support Nonsymbolic Number Sense; projection MR on tangibles maintains attention on the task and might help generalizability; questions about quantity should be related to the narrative goal; eliciting explanations can support children's thinking process, and TUI's provide mindful interactions.

We contribute to the field by 1) presenting important insights about the use of projection-based MR systems by 3- to 5-year-old children; 2) studying on a math concept that has not been implemented with TUIs before and 3) share a detailed design process and insights for future studies. We discuss our results and observations in relation to our future work and underline the potential of MR based tangibles for education and interaction design research**.**

1 BACKGROUND & RELATED WORK

1.1 Nonsymbolic Math

Symbolic math skills are the formal construct of mathematics, such as counting and precise arithmetic operations, such as 3+2=5. Children between the ages 3 and 5 can usually count to 20 and discriminate numerals between 1-10 by age of 5. On the other hand, nonsymbolic math (NSM) ability is the conceptual understanding and judgment between quantities. To put it simply, it is the ability to compare two different amounts as 'more' or 'less' without counting. The difficulty of these comparisons is ratio dependent: 1:2 ratio is easy while 4:5 is hard to compare for the early ages[17]. Several studies assert that comprehension of the nonsymbolic number concept provides a foundation for the symbolic math skills[17,37,48]. To that end, recent interventions that train NSM abilities showed children's progress in comparing ratios as well as how this progress also reflected on their symbolic math skills [6,36,45,46].

These NSM trainings use screen-based applications. However, mathematical development, both symbolic and nonsymbolic, is dependent on interactions with the physical world. Sensory magnitudes (space, area, volume) are present in the natural environment, and approximate number sense may not fully develop without perceiving these sensory magnitudes [43]. Piaget and many other scholars show that children's perception of quantity is affected by space and volume[18,21]. For example, children between 2 and 6 years of age are presented the same amount of marbles in different arrangements. Although the quantities are conserved, children most likely think that their amount has changed. Other studies show that the accuracy of comparing non-numerical magnitudes such as area corresponds with numerical magnitudes as well[24].

1.2 Embodied Influences and Manipulatives

The role of bodily actions on the cognitive processes gathers multiple support from both cognitive science and developmental psychology. Children are active in their learning[47]. Delegators of this constructivist view suggest that knowledge stems from actions, and builds on experience and reflection[61]. Theories of embodied cognition state that individuals make sense of abstract concepts through these physical actions[7,19,30,62]. Also, physical actions, gestures, and objects assist thinking about these concepts [3,22,23], such as counting with fingers to keep track of our mental operation.

Based on the theories explained above, adopting physical manipulatives is a commonly used method to help children understand mathematics[10]. On the other hand, the effectiveness of manipulatives is claimed to depend on the level of perceptual details. Regarding this, there are two viewpoints. The first view holds that detailed, figurative objects help children relate to abstract concepts easier[9]. The other view supports the use of more abstract looking manipulatives by asserting that symbolic objects can inhibit generalizability and captivate more attention to themselves than the educative task [33]. However, simple manipulatives can be too vague for children to relate the concept to the real world [60].

Moreover, handling manipulatives alone might not contribute to comprehension. Studies show that children guided with instruction had higher learning outcomes than children who were allowed for free play discovery[1]. Regardless of the benefits of guided learning, self-discovery is a commonly advocated method as it might support children's own reasoning[16,39]. These points suggest that children need guidance and self-discovery at the same time.

1.3 Tangible and Mixed Reality User Interfaces

Tangible User Interfaces (TUI), as coined by Ishii and Ulmer, merge physical artifacts and environments with the computing paradigm that feature digital responses[57]. The physical and digital couplings are divided into three categories: embedded, discrete, and co-located[50]. Mixed Reality falls under the co-located coupling category (MR), in which object manipulation and digital responses occur within the same display/surface [11]. MR can be implemented with tools such as head-mounted displays (HMD), handheld devices, smart tabletops, and projection systems. Using relatively bulky HMDs may not be suitable for children's use while handheld devices restrain the tangible interaction[34,52]. In smart tabletop applications, computation is embedded into the surfaces and only surroundings can be altered, not object itself[13,15].

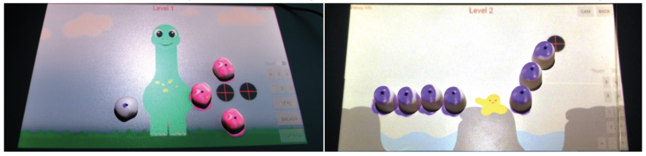

Figure 3. Left: Preliminary design, right: Second version

On the other hand, projection on a surface lays a virtual image on top of a physical object that provides to alter its perceptual properties. SMALLabs uses a room scale technology with a fixed projective system for high school chemistry education[56]. In contrary, we use a tabletop system that enables to focus on tangible interaction. Setups similar to ours, Projection-MR with tangibles, is used for elementary school geography lessons[44], university medical education[29], architectural planning[58], and exploring volumetric data[53]. Additionally, public installations augment objects for playful interactions[26].

In sum, TUI applications for early math development focus on symbolic representations and have not been used for nonsymbolic math yet. On top of the benefits explained in the Embodied Influences and Manipulatives Section, TUIs can provide innovative ways for children to learn in a playful way, combine and recombine the known manipulatives in unfamiliar ways, encourage exploration, support engagement, and imagination[49,51].

To design TUI's for learning, Antle and Weiss provide an extensive guideline based on theories of cognition[5]. Additionally, Hirsh-Pasek et al. present four pillars of learning from the Science of Learning, which is active,

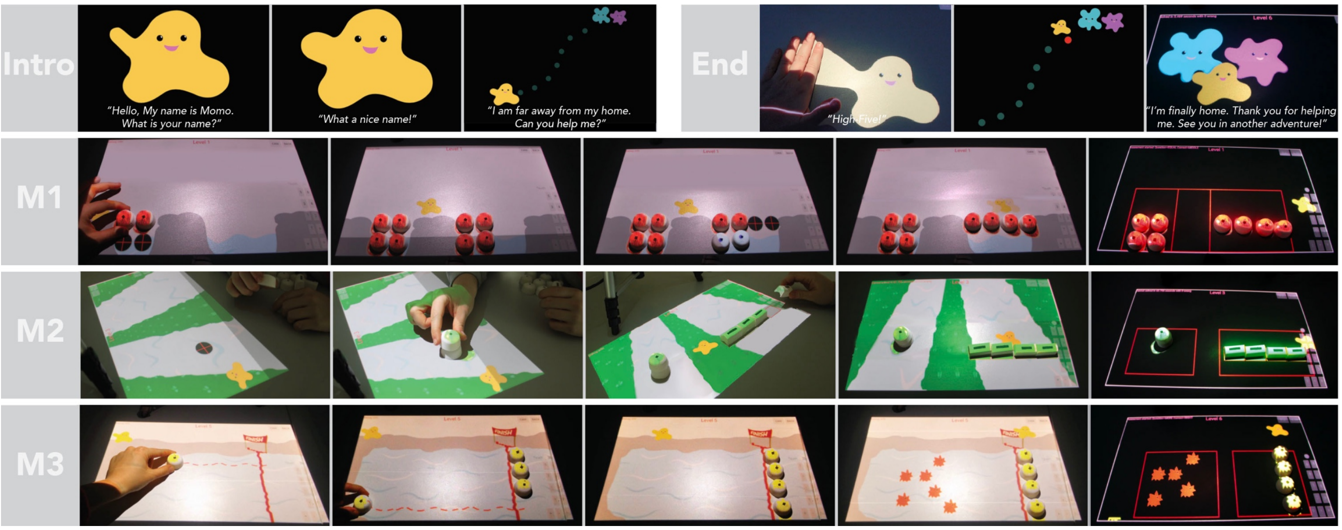

Figure 4. Usage Scenario: Introduction, modules, and ending.

engaging, meaningful, and social learning[28]. We explain how these are reflected in the design of MaR-T.

2 DEVELOPMENT OF MAR-T

2.1 Preliminary Design and First User Study

Our initial design focused on a single training module on approximate amount comparison. The training had a dinosaur image that asked the children to place tangibles on the surface to reveal magical powers. If the physical input was complete, the software would project a different amount on the other side of the surface and the dinosaur would ask which side had more/fewer stones (Figure 3).

We conducted a preliminary study with a total of four children (*Mage*= 3.8 years) [54]. The children were the testers of the system and the sessions were carried out individually[14]. We elaborate further on the findings of this first study in the Designing MaR-T section. In brief, our observations revealed that unrestricted MR setup might encourage children's exploratory behavior, as some of them tried to discover alternate ways of putting tangibles or methods for eliciting responses from the system. Moreover, we observed that fostering multi-modal interactions might provoke a mindful way of interacting with educational tools. That is, the actions during the game are divided into physical input and the gestural answer (pointing) that prevents them from performing repetitive actions such as putting objects only. On the other hand, we also observed the need to integrate further guidance that could improve the MaR-T since some children failed to see why it was incorrect. These preliminary observations showed that projection-based MR holds a promise for this age group's use and pointed at certain aspects that needed refining. We also improved other features which we discuss in the Designing MaR-T section.

2.2 MaR-T: Usage Scenario

In this section, we explain MaR-T's training modules and aims, along with interaction details (Figure 4).

2.2.1 Introduction Children first get familiarized with the genderless and abstract looking character, Momo. (Figure 4) Momo introduces itself, asks the child's name, and compliments on it. Then it asks help from the child to go home. After the child says that she/he will help, Momo shows where it is on the map. Between the levels, the child sees Momo's progression on the road to see how many levels are left. Momo asks for a high-five from the child to motivate her/him before moving on to the next level.

2.2.2 Module 1, conservation of quantity. This module aims to train that spatial reconfigurations do not change the quantity of tangibles. To start, Momo asks the child to put the objects at two sides of the surfaces' designated spots which are duplicates to each other. After Momo crosses the first cliff, it realizes that the next layout cannot work. The child rearranges the same objects in a different spatial layout as shown (Figure 4, M1). In the end, the child sees two equal amounts of objects on both sides of the surface, but in different arrangements. Momo is then curious to learn if one side has more objects (or less, depending on the question) or if the amounts are the same. To answer, Momo tells the child to point to the side that has more objects or to put her/his hand in the middle if they are equal. Upon the response, Momo asks for an explanation. The surface displays a trophy and plays audio to signal accomplishment that is repeated in each correct answer. If the answer is incorrect, Momo advice the child to reconsider and repeats the question. If the child still answers incorrectly, the level passes without a trophy visual.

2.2.3 Module 2, Comparing amounts in different axes. The aim of this module is to make children experience that different volumetric characteristics and axial layouts (stacked) of objects do not affect their numeric quantities. In the beginning, Momo stands in front of two rivers. For the first river, Momo directs the child to put objects on top of each other to hop on this stomp. For the other one, the water is less deep but requires a road to cross it. The child puts rectangular stones side by side in the designated spots. Then, the question cycle is the same as the first module.

2.2.4 Module 3, Comparing approximate amounts. This modules goal is to make children compare two amounts without counting. In the beginning, Momo prompts the child to put an object on the desired areas. After placement, a creature appears on the object. Momo tells the child that this friend needs help swim across. The child slides it to the other side of the surface. After all creatures (objects) are on the other side, another set of creatures appear. The child is told to look with attention because these creatures can hide very quickly. We specifically remove the visuals only in this module since the approximate comparison is done without counting [25]. Afterwards, the child again compares the amount as more/less (asked in randomized order) and explains her/his answer to Momo.

2.2.5 Ending. The child sees that Momo reaches home and is reunited with its parents. Momo thanks the child and wishes to see her/him again.

2.3 Designing MaR-T Based on Theories of Cognition & Learning

In our first study, we saw that children were able to interact with the system, along with the positive findings and the need for refinement. To enhance our training effect, we adopted the four pillars of learning (active, engaged, meaningful, social) in a systematic way. In this section, we present how these pillars, TUI guidelines, and the age group's needs helped us develop our design. We explain the changes by comparing it to the first version of MaR-T.

2.2.1 Active Learning is defined as being 'minds on' with a task, not just making redundant actions but exerting mental effort [28]. TUI's can be designed with spatial, physical, temporal or relational properties to slow down interaction and trigger reflection[5].

In our first study, we saw this kind of mindful interactions that support active learning. Our preliminary game required different actions in each step: 1) placing all the objects, 2) assessing the consequent projected ratio, and 3) gesturing while giving an answer. As each step was connected, the children focused on each action in order to play. So, we put specific attention to keep these properties in the second prototype as well.

We improved MaR-T further by having Momo ask reflective questions to children, such as "Why do you think that this side has more?" to make them contemplate more about their selections. Also, to relieve mental operations about judging a quantity, our objects were designed to allow spatial reconfiguration to help children think in a 'hands-on' manner while explaining their answers [5].

2.2.2 Engagement pillar stands for staying focused on the task. Providing responsive interactions, extrinsic and intrinsic motivation support engagement. TUI's already provide responsive interactions and extrinsic motivation with feedback[51]. On the other hand, intrinsic motivation can be sustained through curiosity. Bekker et al. note that the combination of novelty, complexity, and uncertainty elements in a design sustains such curiosity[55].

In our first study, the children were excited and engaged by the novelty of projection MR. However, we thought that placing objects and eliciting responses would become familiar and less engaging in time. To amplify curiosity, we followed Bekker et al.'s combination by adding different physical interactions. Out of various interaction possibilities, we chose three actions congruent to our training aims: placing, stacking, and sliding (Figure 3). Distance manipulation is used for conservation of quantity tasks via placing and re-placing objects while stacking for comparing quantities in different axes. Sliding was chosen to break the redundancy of placing objects by provoking the uncertainty that keeps children on their toes.

2.2.3 Meaningful Learning occurs if we connect our existing knowledge with a new concept and is reflected when the child can extend the knowledge to different situations. Providing a narrative contingent with the taught domain may help build relevant comprehension.

In our first study, we only had one module that focused on the approximate comparison. In order to target children's confusion between volumetric magnitude and numerical magnitude, MaR-T has two more modules with various spatial arrangements. Moreover, MaR-T follows a narrative applicable to the concept of quantity such as placing objects on the surface to pave a road/bridge/stairway.

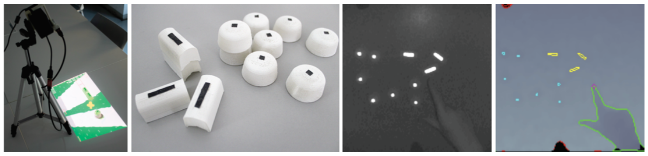

Figure 5. From left: The setup, objects, IR & depth images

2.2.4 Social Interaction can be a key component in learning. It is shown that familiar social agents enable children to learn better from than those who are devoid of social contingency. Parasocial interaction techniques such as talking to the onlooker and demanding responses foster a social contingency that contributes to learning[31].

The first study featured a character to guide children but did not have any animated characteristics or dialogs with the children. We had observed that children paid limited attention to it. To that end, MaR-T introduces a social character. The child interacts with it verbally and physically (high-five). These parasocial elements are further supported via dialogs and feedback (Figure 3a & 3b).

2.2.5 Guided Exploration Hirsh-Pasek et al. suggest scaffolded exploration towards a learning goal. Open-ended play is fruitful for engagement, exploration, and suited for supporting creativity. However, a structured exploration supports learning better than an open-ended play[28].

Same as our first study, we provide specific locations for children to put the objects. This is done for two reasons, first, if children focused on exploration a lot, learning can be interrupted. Second, we control the distances between objects to make them compare specific arrangements.

2.2.6 Needs and Abilities of Pre-K Children, ages between 3 and 5 have cognitive, social, emotional, and social abilities that demand appropriate designs [8,59]. Instructions need to be step-by-step, equip additional support and repetition, and tasks need to be introduced one at a time. We paid close attention to these considerations in verbal feedbacks. We also added the repetition of questions in the second version in MaR-T, which was lacking in the first version.

Moreover, Self-esteem of this age group needs rewards via positive feedback. For this purpose, we provide verbal complements, audio effects, and visuals. This age group is developing empathy, which we targeted by helping the character reach home. Further, their fine motor skills are developing and usually practice gripping via object like blocks. The design of MaR-T tangibles is discussed below.

2.2.7 Tangible Objects We designed 3D printed amorphous white tangibles with smooth surfaces. This was to minimize the attention drawn their physical properties and to enable a clear projection of visuals (Figure 5). The rough diameter and height are 30mm, suited for the age group's hand grip. Different than our previous study, we placed magnets into the tangibles to help stacking.

In sum, we aimed to promote *active learning* by asking thought-provoking questions, added new physical interactions to maintain *engagement*, added different spatial reconfigurations in each module towards *meaningful learning*, and added parasocial relations with our character to build *social interaction* according to the needs and abilities of pre-k children.

2.4 Technical Details

2.4.1 Hardware We use a mobile Android (version 5.1.1) device designed in our lab. Instead of a touch screen, it has a Sony CXN020X pico-projector as a display with 1280x720 resolution. It is equipped with Qualcomm APQ8016 (CPU: 1.2 GHz) processor and 924 MB RAM.

Also, we connect an external ToF(time-of-flight) depth camera, PMD CamBoard pico flexx, that can give 224x171 point cloud and IR (infrared) image in 0.1m – 4m range. We mount the device onto an adjustable tripod (Figure 5). The setup can be relocated easily and does not require any installation.

2.4.2 Software Implementation. The game is developed in the Android environment with native components for object and gesture detections. We used OpenCV[66]for image processing and RoyaleSDK[67] of PMD to get 3D point cloud and IR image. Lottie[68] and Bodymovin[69] libraries are used to create and display animations. All the calculations are done in the device's CPU.

In MaR-T, the depth camera and the projector have different resolutions and field of views. Since they do not share the same coordinate space, the transformation between them needs to be calculated beforehand. To estimate this transformation, we project a little circle onto a surface, then we put a retro-reflective marker on it and capture the 3D location of the marker from the depth camera. This procedure is repeated many times for different surface planes to obtain necessary 2D-3D location pairs. Then we estimate the transformation by using least-squares regression. Performing this calibration once is enough since the relative positions of the camera and projector do not change even if the system is relocated.

Since our processing capacity is limited, we kept the tracking procedure simple. We put retro markers on top of each stone (Figure 4). Binary thresholding and blob detection are applied to the IR image. The 3D locations of the blob centers are acquired from the depth image and they are converted into 2D pixel locations in projector space by using the transformation we estimated before. Orientations of the rectangular stones are also calculated from the shape of the detected blobs.

To capture the pointing gestures, we apply a similar technique used in[63]. The first time we place the setup, 20 consecutive depth images of the empty surface are taken, and their average is used as the background image. In each frame during the game, the background image is subtracted from the current depth image. To determine hand contours, binary thresholding followed by median filtering is applied to the resulting difference image. Contours that do not touch one of the image edges are discarded. The topmost points of the remained contours are determined as the pointed location and they are mapped to projective space by using estimated transformation.

3 METHODOLOGY OF THE USER STUDY

MaR-T was developed in an iterative process where the role of the children as testers contributed to the design of the system [14]. They were informed by the researchers that they would play a game and their opinions about it would help make the game better [4]. The participants were observed during the interactions with regards the points they enjoyed, struggled with, or experienced confusion. As the literature suggests, this age group might be motivated to please the adults (researcher) and can experience hardship expressing their true feelings with questionnaires or scales [27,38]. To that end, the researcher conducted a semi-structured interview and asked about what they did during the game and what part of the interaction they liked/disliked [14]. We also discussed each children's performance and behavior with their teacher to interpret the findings further. When closing the session, the researchers thanked children for their contribution [4].

3.1 Participants & Research Setting

Ten children from a local kindergarten participated in our study and were invited to the session one by one. The participant's age average was 4.7 (SD= 0.483, 7 females, 3 males). The study was conducted in a room in the kindergarten to help them feel more comfortable [35]. The two researchers were present with the child during the interactions and only provided occasional guidance to minimize the influence on natural interactions. One researcher handled the setup with her laptop in the corner, the other one sat next to the child. Each session was videotaped, as approved by the parents. The sessions lasted 10 to 15 minutes for each child. Details about the recruitment process can be seen in the last section.

3.2 Data Analysis

Children's answers to the math questions were recorded by our setup's software. The interviews were transcribed and coded. During video analysis, for meaningful learning, we paid attention to children's tangible interactions and how these affected their responses. We observed whether if children gave explanations to their answers for active learning. For assessing engagement, we coded facial expressions (frowning, smiling, etc.), exclamations ("Wow", "Pff" etc.), posture, gaze and focus on the task. We classified them into four levels of engagement: high engagement (enthusiastic face with eyes focused on the task, upright posture), low engagement (neutral face, focused on the task), low disengagement (staring around occasionally, slightly glum expression, changing body posture frequently), and high disengagement (glum facial expression, non-responsive behavior towards the instructions, leaving the task) [64].

4 FINDINGS & DISCUSSION

All children completed the three-module training session. The incorrect answer distribution and engagement levels of the participants can be seen in Table 1. The semi-structured interviews revealed that the children liked the game and five of them wanted to play again. Their detailed opinions and comments are disclosed in the relevant headlines. We discuss all the findings and their implications below.

4.1 TUI's physical affordances might support Nonsymbolic Number Sense and meaningful learning

We had expected that the spatial characteristics of tangibles would change children's perception of quantity [21,43]. Apparent in Table 1, all four-year-olds had misconceptions in the conservation of quantity (module 1) and two had incorrect answers while comparing the amounts in different axis' (module 2). They had the tendency to think that sensory characteristics such as spatial arrangement and their shape affected their numerical quantity (i.e., long objects mean more, stacked objects are less). This further supports our argument that training with tangibles is necessary to overcome this confusion between sensory and numeric magnitudes. In the next iteration, we will accompany incorrect answers with Momo's explanations.

This finding also suggests that training nonsymbolic math via screen-based applications can lack meaningful learning. As children's misconceptions related to three-dimensional features will not be realized/addressed, they might be unable to use their knowledge correctly in the real world.

On the other hand, five-year-old participants were successful in the first two modules and their mathematical development was advanced, as we confirmed with their teachers. The results indicate that four-year-old children can be recruited for training, yet we will have pretests with all participants to see their nonsymbolic math knowledge. In our future longitudinal studies, the target group will be naturally those who have difficulties in non-symbolic math comprehension. Moreover, there were incorrect answers in the last module [Table 1]. The last module's quantities disappear in seconds to prevent counting[25] for approximate comparison, but for 4 participants out of 10, this only pushed them to count faster and fail. To prevent this, we are planning to increase the objects numbers, and give clear directions to not to count.

Table 1. Participants engagement levels and answer distribution

Participants	Age	Engagement Level	Module 1 *Conservation of quantity*		Module 2 *Amounts in different axis'*		Module 3 *Approximate amounts*	
P1	5	High Engaged	+	+	+	+	+	+
P2	5	High Engaged	+	+	+	+	+	+
P3	4	Low Engaged	-	-	+	+	+	+
P4	5	Low Disengaged	+	+	+	+	+	+
P5	5	High Engaged	+	+	+	+	+	-
P6	5	Low Engaged	+	+	+	+	+	+
P7	5	High Engaged	+	+	+	+	-	-
P8	5	High Engaged	+	+	+	+	+	+
P9	4	Low Disengaged	-	-	-	-	-	-
P10	4	Low Engaged	-	-	+	-	-	-

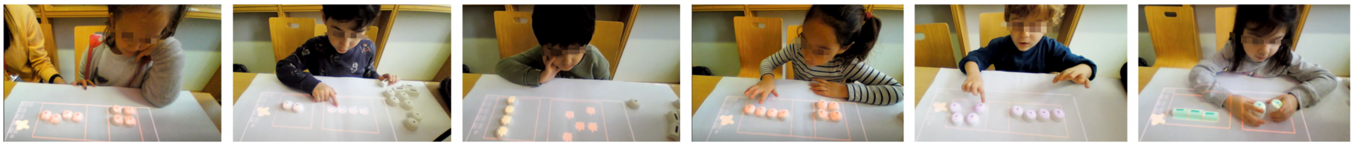

Figure 6. Participants contemplating on their answers and explaining them.

4.2 Projection MR on tangibles maintains attention on the task and might help generalizability of the taught concepts

None of the participants were distracted by the projections on the tangibles, i.e. examined the visuals briefly. This supports our assumption that projection can keep the attention on the task rather than the tangibles. We also observed that one child had difficulty in distinguishing the projection on the stone in Module 3. Designers should consider the background color, projection saturation, and lighting conditions carefully.

After each study, the experimenter asked the participants to explain what they did during the game. Five of the participants said that they played a game with stones; three participants stated that they played with Momo, and one child refused to provide an answer (see section 4.7.3). Only one participant commented on the use of tangibles for multiple purposes and stated, "I made some friends swim across and built roads to help Momo go home." This explanation is encouraging towards using MR tangibles to support generalizability, yet we cannot make such claims due to a limited number of sessions and participants.

4.3 Questions about quantity should be related to the narrative goal

The question about comparing the quantities proved to be problematic for two reasons. After children placed the objects onto the surface and Momo passed the road, two participants thought that they passed the level and moved the tangibles away. The comparison question took them by surprise, and they had hard time answering. For them, their goal was helping Momo cross and the question seemed irrelevant. Second, two participants, even if they had the tangibles in front of them, felt pressured and thought of Momo's question as a test. We should also note that helping Momo understand which side had more/less did motivate the rest of the children.

These findings suggest that the questions about quantity should be asked towards a goal, such as picking the side that has more 'apples' to satiate the character. This way the children would be motivated to choose quantities and feel more at ease.

4.4 Eliciting explanations can support children's thinking process and active learning

Different then our preliminary study, we asked reflective questions to children about their answers. Nine children (out of 10) gave descriptive answers, which indicates that they were 'minds on' with the task. Children's verbal and gestural explanations also revealed their thought processes and the points that they struggled (Figure 6)[3]. For two participants, explaining enabled them to realize their incorrect answers, which is crucial for active learning.

While designing training for children, reflective questions can support minds on behavior and their explanations create a remarkable opportunity to examine their knowledge level. We are planning to develop MaR-T not only to monitor children's processing but also to tailor the training according to each child's needs, which is a technological capability that most studies overlook[41].

4.5 TUI's provide mindful interactions for engagement

As suggested by Antle et al., our setup supported spatial, physical, temporal, and relational properties that contributed to slowing down interactions. We explicitly observed the benefit of this mindful interaction in one highly active participant. During the first trial, he did not comply with the instructions but placed and threw objects as he desired. Once he could not draw any reaction, he followed the guidance and slowed down his interaction with tangibles. The positive responses engaged him, and he continued in a calm manner.

This mindfulness caused by the TUI setup contributes to active learning and engagement. This is not only vital for learning but also to help children comply with instructions to prepare them for formal education[42].

4.6 TUI's can adopt different interaction mechanisms to sustain engagement

In our first study, the children only placed objects and pointed at the answers, which we thought would start becoming less engaging in time. Therefore, in the second iteration, we considered several interaction possibilities with tangibles to keep curiosity alive (novelty-complexity-uncertainty loop [55]). All participants were hooked by the first novelty effect, apparent in their gasping and wide-eyed responses. As different interactions were introduced in each module, we observed curiosity in participants facial expressions (i.e., furrowed brows) and they asked Momo what to do next. Momo's guidance helped the children resolve this uncertainty. However, we observed that one child got quickly frustrated, since, for him, Momo was slow to give a clue (Figure 7, c). Thus, we acknowledge that there is a thin line when providing new interactions, such as making it easy to figure out and providing the clues more quickly to prevent frustrations.

Different from our preliminary study, we observed that sustaining curiosity [55] with interactions has the potential to keep the children engaged with the training.

4.7 Parasocial relations contribute to children's social interaction and engagement

In our preliminary design, we had featured an image of a character that only provided guidance and feedback. This time, Momo was designed as an animated 'social' character and the impact of this change was very positive.

4.7.1 Parasocial interaction supports following instructions.

In our preliminary study, the children tended to ask researchers what they should do and did not listen to the characters instructions carefully. In MaR-T, starting with an Introductory dialogue ("What is your name? -What a nice name!") showed participants that Momo was a social character, which made them pay close attention to its guidance. They waited for Momo's cues to proceed and did not ask any questions to the researchers. Moreover, we observed that all children were smiling and talking directly at Momo throughout the trials (Figure 8). Giving high-fives to the character induced giggles all around.

4.7.2 The narrative provides extrinsic motivation and emotional relevance Our first study did not have a deliberate narrative goal. We had presented the training just as a game. In MaR-T, children were playing the game to help Momo reach its home. This created empathy and the children were eager to help the character. Moreover, after each level, the children saw their progress on the roadmap, which provided a sense of accomplishment and further excitement. One child exclaimed, "Momo is almost home!" Once Momo reached home and reunited with its parents, all children looked happy.

4.7.3 Engagement signs vary with the character of the child

According to their teachers, the participants who were low engaged and disengaged were shy. These children showed reluctance to make eye contact and to talk with the researchers before the session. They were more responsive after they started playing with the character and provided explanations. Yet, they showed low signs of positive emotions (smiling faintly) and displayed a passive posture.

The only low disengaged participant did not give any explanation to his responses. He interacted with the character by telling his name, smiled and said yes to helping Momo. He placed the objects in the shown spots but gave wrong answers without explanations. The researcher, only once, tried to probe him to reflect on answers, which revealed his counting errors.

We think that parasocial relations were supportive of these children only to an extent. Designing more supportive feedbacks such as "Let's look at the tangibles together, let's count!" can be helpful to relieve their stress.

5 LIMITATIONS & FUTURE WORK

MaR-T's target group is 3- to 5-year-old children. However, we had trouble recruiting 3-year-olds. We will have further sessions with younger age groups to reveal their needs as well. We were not able to assess meaningful learning based on one session. For future work, we will have longitudinal training sessions with children to see the effectiveness of our setup for nonsymbolic math. We are planning to have control

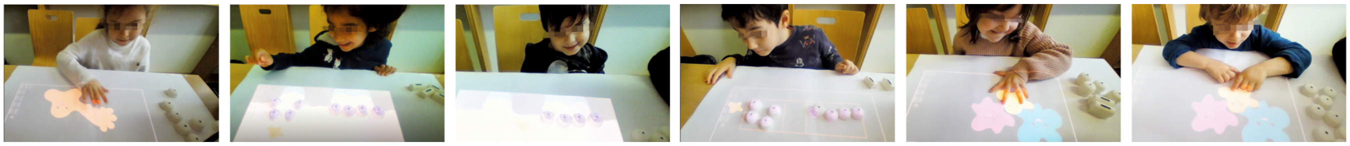

Figure 7. Children interacting with Momo

groups with tablet condition and develop measurement methods to assess math development.

For a projection setup, ideally, direct lighting should be avoided on the surface to see the visuals more clearly. We covered the top of the setup as we could not control the light coming into the room.

In the next iteration, we will have the narrative provide explanations in the case of wrong answers to train the children thoroughly. Moreover, we will also refine the points discussed in the previous section.

6 CONCLUSION

In this paper, we presented MaR-T, a projection-based MR system that aims to train 3- to 5-year-old children's nonsymbolic number representation through tangible interaction. We contribute to the literature by 1) studying a math concept that has not been implemented with TUIs before, 2) exploring the potential of projection-based MR setup with tangibles for preschoolers, and 3) a detailed design process and insights for future studies. We conducted user studies to observe the interaction capabilities of the setup and assessed our design choices towards the four pillars of learning (meaningful, active, engaged, social) that are crucial for the effectiveness of our future training sessions.

Towards *meaningful learning*, we need to have longitudinal training sessions to come to a judgment, which is applicable for all pillars. Yet, these findings indicate that training with tangibles might overcome children's confusion between sensory and numeric magnitudes. For this pillar, we also realized the need to integrate quantity related questions into the narrative goal. MaR-T can provide generalizability by augmenting the same object with different analogies contrary to regular tangibles. For *active learning*, we observed that reflective questions make children think further on their choices and reveal their thought processes. Contributing to this pillar, MaR-T can provide a mindful playing environment with its spatial, physical, temporal and relational properties. For *engagement*, we employed a narrative and different physical interaction with tangibles which supported a majority (8/10) of the participants' engagement. For *social interaction*, parasocial relations were established with each participant and were a vital element to help children build social relevance and focus on the task more eagerly.

To conclude, the design process of MaR-T employs research through design method[65], which can guide other designers and researchers who want to apply a similar approach. Additionally, our integration of the TUI guidelines with the four pillars of learning can start the discussion in this education topic and create an important base for the future works. We plan on having longitudinal training sessions with more children and control groups with a tablet condition to deduce our system's specific contribution to nonsymbolic math understanding. If our training will be measured as effective, we think it will be a significant finding for nonsymbolic pre-K math education.

7 SELECTION AND PARTICIPATION OF CHILDREN

Ethical approval for this study was obtained from the authors' university's Committee of Human Research. To conduct our study, we contacted various kindergartens in authors' city. For those who had a positive response, we had meetings with the teachers in detail. Upon providing the teachers with the documents explaining the project and the informed consent forms, they contacted the parents. One parent also phoned us for further information. Other than informing the parents, the teachers also talked to the students about how they had the chance to participate in research if they wished [4]. This created excitement amongst the children and had a great impact on the signed return of the consent forms. Furthermore, we explained the aim of the user study (gaining their insights) to children before starting the session and said that they were free to go back to their classroom if they desired[4]. None of these children quit the session or showed stressed behavior.

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